

Mathematical Formulation of a Model for Landform Attributes Representation for Application in Distributed Systems

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Abstract. In this work we presented a methodology based on non linear regression for Landform Attributes representation. The equations to estimates the parameters of a two dimensional polynomial are shown and, for testing the methodology, it was used the data of Landform Attributes from the state of *Minas Gerais* (Brazil) obtained by the Digital Model of Elevation (DME) from GTOPO30 [5] project, in the form of a regular grid, with spacing of approximately 900m. The presented methodology can be used to minimize time in sending Landform Attributes information through network, to minimize space by storing the parameters of the estimated function and to make possible the distribution of the processing in different CPU, through internet network, for coefficients calculations.

Keywords: mathematical modeling of Landform Attributes, distributed processing, Internet, non linear regression, polynomial estimation.

1. Introduction

The increased efficiency and reliability of computer networks, as well as the popularization of the Internet, generated new opportunities for development of applications. Now, with the growing number of connected computers in the Internet [1][2], it is possible to develop distributed applications, using its communication infrastructure, without great efficiency problems. An application that has access to the resources of the Internet can use the services as HTTP servers, SMTP, POP3, FTP, to updating through remote servers.

In Geo-informatics, there is an area called Numeric Modeling of Land (NML) that treats the mathematical representation of land [3]. One of the numeric models most common used is the Regular Grid Model, which function is to generate a grid starting from a group of altitude points, longitude and latitude as in **Fig. 1**.

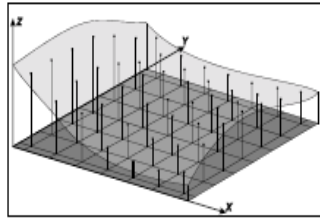


Fig. 1 – Regular Grid Model

This grid can be generated by points interpolation for a polynomial regression model, to adjust a two-dimensional polynomial, which best represents the landform attributes of an area. The regression analysis is a statistical instrument generally used in science. Its common use is making possible the description of a phenomenon through mathematical model (equation), based on a data sample. Graphically, it is equal to identify the curve or mathematical surface that better adjusted to the points in the dispersion diagram. The mathematical models of regression are based in three statistical assumptions: a) the relationship among the dependent and independent variables is deterministic instead of stochastic; b) the errors measurement are random, following the Normal Distribution, average zero and constant variance; and c) the explanatory variables don't show correlation among themselves [11]. This technique of landform attributes representation has some advantages over other techniques, basically by representing the landform attributes through mathematical equations instead of images, latitude coordinates, longitude and altitude, or map of altitude levels. One of the advantages is the significant reduction of the amount of necessary information to represent the landform attributes of a certain area, since with the regression technique, the landform attributes of an entire area, can just be represented through the coefficients of two-dimensional polynomial. Through the polynomial representation it is possible to generate images with different resolution levels, since the data come from an image and the number of points used for its representation. Besides these advantages, there is a possibility to apply, on the polynomial, several mathematical functionalities, such as, how to find the maximal and minimal point, derivation, etc. However, the technique of polynomial regression has some disadvantages, since the complexity and the computational power demanded in obtaining such polynomial is very high and sometimes impractical.

For this reason the objective of this article is to present a methodology designed to make possible the distribution of the necessary processing to compute a two-dimensional polynomial that represents the landform attributes of an area and, as an example, we use the area of the state of *Minas Gerais*, in Brazil, located in the Southeast Area of the country. Some estimative calculations, presented in this article, show that the necessary time, of centralized processing, to estimate such a polynomial is prohibitive, being in order of dozens of uninterrupted years of processing.

2. Mathematical Method

In this work we presented a methodology for Landform Attributes representation using the method of non linear regression to adjust a two-dimensional polynomial. The regression analysis is a statistical instrument very used in the science, in general. Its great use is due to the fact of making possible the description of phenomena through mathematical models from a data sample. Graphically, it is equal to identify the curve or mathematical surface that better is adjusted to the points in the dispersion diagram.

The mathematical models of regression are based in three statistical facts: a) the relationship among the dependent and independent variables is deterministic instead of stochastic; b) the errors are random with normal distribution, average zero and constant variance; and c) the explanatory variables don't present correlation among themselves [4].

When a mathematical model of regression is used, the method of estimate the parameters most used is the minimum square method, which consists in estimating a function to represent a group of points, minimizing the square of the deviations [?]. Considering a group of geographic coordinate (x,y,z), taking an estimating altitude function $\hat{z} = f(x, y)$ of these points, a polynomial of degree **r** in **x** and degree **s** in **y** can be given, according to the **Equation 1**, and the estimated error ε_{ij} given by the **Equation 2**, where $0 \leq i \leq m$ and $0 \leq j \leq n$.

$$\hat{z} = f(x_i, y_j) = \sum_{k=0}^r \sum_{l=0}^s a_{kl} x_i^k y_j^l \quad \text{Equação 1}$$

$$\varepsilon_{ij} = z_{ij} - \hat{z}_{ij} \quad \text{Equação 2}$$

The coefficients a_{kl} ($k = 0, 1, \dots, r, l = 0, 1, \dots, s$) that minimize the errors of the estimated function $f(x,y)$, can be obtained by solving the **Equation 3** for $c = 0, 1, \dots, r$ and $d = 0, 1, \dots, s$.

$$\frac{\partial \xi}{\partial a_{cd}} = 0 \quad \text{Equação 3}$$

Where

$$\xi = \sum_{i=1}^m \sum_{j=1}^n \varepsilon_{ij}^2 = \sum_{i=1}^m \sum_{j=1}^n (z_{ij} - \hat{z}_{ij})^2$$

and

- x_i Longitude i of MDE column, for $1 \leq i \leq k$
- y_j Latitude j of MDE line, for $1 \leq j \leq l$,
- z_{ij} Altitude of (x_i, y_i) point
- r** Polinomial degree in x,
- s** Polynomial degree in y.
- a_{kl} Coefficients which minimize the error of the estimated function $f(x,y)$

By solving the **Equation 2** we get the **Equation 3** and, by solving the **Equation 3**, we get the **Equation 4** through **Equation 10**.

$$\varepsilon_{ij}^2 = \left(z_{ij} - \sum_{k=0}^r \sum_{l=0}^s a_{kl} x_i^k y_j^l \right)^2 \quad \text{Equação 4}$$

$$x_{lc} = \sum_{i=0}^m \sum_{j=0}^n x_i^{l \operatorname{div}(s+1)+c \operatorname{div}(s+1)} y_j^{l \operatorname{mod}(r+1)+c \operatorname{mod}(r+1)}$$

Equação 11

$$b_l = \sum_{i=0}^m \sum_{j=0}^n z_{ij} x_i^{l \operatorname{div}(s+1)} y_j^{l \operatorname{mod}(r+1)}$$

Equação 12

The **Equation 11** and **Equation 12** can be calculated in an independent way, fundamental characteristic for the distribution of the computations, making possible the distribution of processing in different CPUs over a network system. The complexity function for **Equation 11** and **Equation 12** is given by **Equation 13** and **Equation 14**, respectively.

$$n(r+1)^2(s+1)^2 \approx O(n^4)$$

Equação 13

$$n(r+1)(s+1) \approx O(n^2)$$

Equação 14

3. Testing Data

To validate the presented methodology and equations, they will be applied to represent the Landform Attributes of the area of the state of *Minas Gerais* (Brazil). The data source of the chosen area, comes from Digital Elevation Model (DEM) from the GTOPO30 [5] project, in the form of a regular matrix, with 1043 lines and 1343 columns, with spacing approximately 900m in the geographical coordinates.

4. Results and Conclusion

Using the data source form the GTOPO30 [5] project, the statistical analyses of the altitudes of the state of *Minas Gerais* indicate a dispersion from 1m (meter) to 2.863m (meter). The distribution of these points, in 100m by 100m intervals, is presented in **Table 1** below.

Table 1 - Classes of altitudes and relative percentage of the points, in each class, considering the DEM data of the State of *Minas Gerais*. The classes are intervals of 100m, from 0 to 2.900m.

Classes of altitude (m)	Number of points	Percentage of point in a class (%)	Accumulated percentage of points at classes (%)
0001 - 100	227	0,03143	0,0314
0101 - 200	4828	0,66843	0,6999
0201 - 300	13674	1,89313	2,5930
0301 - 400	35192	4,87225	7,4652
0401 - 500	33607	4,65281	12,1181

0501 - 600	94169	13,03749	25,1555
0601 - 700	169517	23,46925	48,6248
0701 - 800	80245	11,10974	59,7345
0801 - 900	89571	12,40091	72,1354
0901 - 1000	136910	18,95489	91,0903
1001 - 1100	18658	2,58316	93,6735
1101 - 1200	17588	2,43502	96,1085
1201 - 1300	20127	2,78654	98,8951
1301 - 1400	2207	0,30555	99,2006
1401 - 1500	1923	0,26624	99,4668
1501 - 1600	2559	0,35429	99,8211
1601 - 1700	406	0,05621	99,8773
1701 -1800	405	0,05607	99,9334
1801 -1900	281	0,03890	99,9723
1901 - 2000	50	0,00692	99,9792
2001 - 2100	50	0,00692	99,9862
2101 - 2200	49	0,00678	99,9929
2201 - 2300	15	0,00208	99,9950
2301 - 2400	9	0,00125	99,9963
2401 - 2500	10	0,00138	99,9976
2501 - 2600	6	0,00083	99,9985
2601 - 2700	7	0,00097	99,9994
2701 - 2800	3	0,00042	99,9999
2801 - 2900	1	0,00014	100,0000
Σ	722.294	100,00000	-

The DEM, with 1400749 points, has only 722294 points belonging to the state of *Minas Gerais*, the others points are outside the state area. Using all the points representing the Landform Attributes of the state and by using **Equation 11** and **Equation 12** we estimate a polynomial coefficients for representation the Landform Attributes of the state of *Minas Gerais* for $r = s = 2, 3, 10$ and 20 . The processing was done by a Laptop Asus computer M5200AE, running Windows XP on Intel Pentium M Processor 740, with 1.73 GHz and 512MB RAM. The time values for processing in second $t(s)$ and the regression coefficient R^2 of the adjusted polynomial of degrees r in x and s in y , with $r = s$, are presented in the **Table 2**.

Table 2 – Time processing in seconds $t(s)$ and R^2 for the adjusted polynomial of degree r in x and degree s in y , for $r=s$.

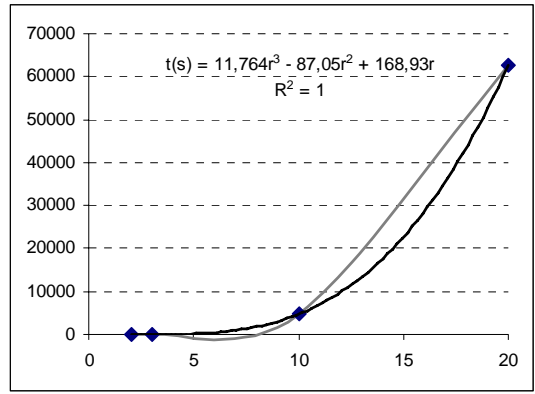
$r=s$	$t(s)$	R^2
2	28	0,41547
3	86	0,47168
10	4746	0,60450
20	62675	0,64767

With the values from the **Table 2** and by using polynomial regression, we get the **Equation 15** and **Equation 16**, to estimate the time (ts) and R^2 , respectively, with $r=s$.

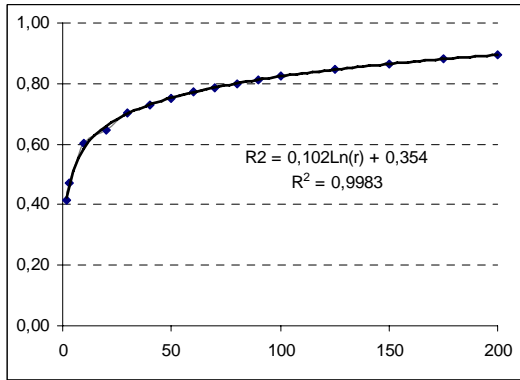
$$t(s) = 11,764s^3 - 87,05s^2 + 168,93s, r^2 = 1,0$$

Equação 15

$r^2(s) = 0,102 \ln(s) + 0,354$, $r^2 = 0,9983$	Equação 16
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(a)



(b)

Fig. 3 – (a) Processing Time estimative; (b) Estimated error

From **Equation 15** and **Equation 16** we simulated the time processing in second $t(s)$, **Fig.3-a**, and the coefficient of regression R^2 , **Fig. 3-b**, for the adjusted polynomial which represents the Landform Attributes for $r=s$, varying from 30 to 200 degree.. As an example, for $r=s=500$, the model of Landform Attributes representation, for the state of *Minas Gerais*, is adjusted with $R^2=0,988$ and we need 18.4 years to generate the coefficients that solve the matrix system above (**Fig. 2**)

Table 3 – Processing time in seconds (ts) and R^2 for the polynomial of degree r in x and s in y , $r = s$, from 30 to 500 degree.

$r=s$	$t(s)$	R^2	$r=s$	$t(s)$	R^2
30	244351	0,70102	225	129630416	0,9065
40	620373	0,73037	250	178414108	0,9173
50	1261322	0,75313	275	238117737	0,927
60	2237780	0,77172	300	309844179	0,9359
70	3620332	0,78745	325	394696309	0,9441
80	5479562	0,80107	350	493777001	0,9516
90	7886055	0,81308	375	608189130	0,9587
100	10910393	0,82383	400	739035572	0,9652
125	21637523	0,84659	425	887419202	0,9714
150	37770215	0,86518	450	1054442894	0,9772
175	60411344	0,88091	475	1241209523	0,9828
200	90663786	0,89453	500	1448821965	0,988

Using the DEM data source, an altitude map was generated in gray tones, **Fig. 4-a**, were can be observed that state of *Minas Gerais* has a heavily uneven topography. With the coefficients of the adjusted polynomial of degrees $r=s=20$ the altitude map were also generated, **Fig 4-b**. In **Fig 4-c** we show the relative errors computed using the source data (DEM) and the estimated data (polynomial).

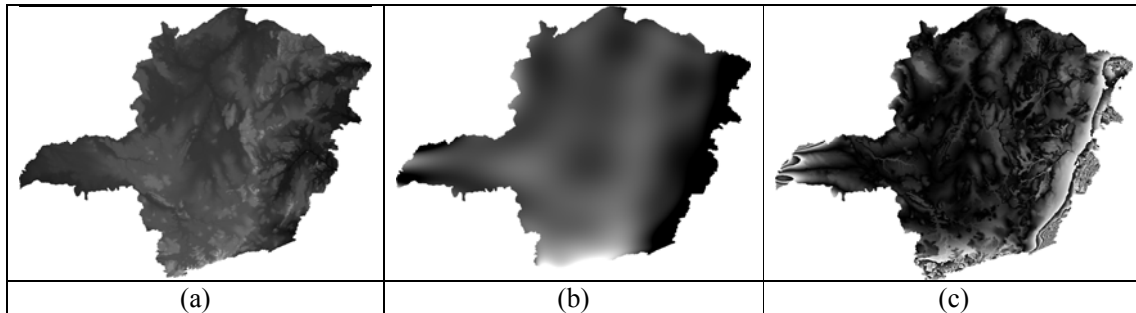


Fig. 4 – Altitude map of *Minas Gerais* state in gray tones (a) from de DME source (b) form the estimated polynomial with $r=s=20$ (b) and (c) the relative errors form the data source and the estimated dada. Dark gray represents less altitude.

Since we used only points that represent the Landform Attributes of the state of *Minas Gerais*, the bordering areas suffered heavy alterations, generating emphasizes not existent. That anomaly can be corrected taking, for estimation of the polynomial parameters, points that cross the limits of the real area.

The altitudes of DEM and the altitudes estimated by the polynomial of degrees $r=s=20$ are displayed in classes of 100m in **Table 4**. By analysis of these data we may verified that the class data are comparable, although the adjusted polynomial generates some values above the maximum and minimum altitude. It is believed that with larger r and s , we can reach better values of R^2 and this will improve the accuracy of the polynomial parameters.

Table 4 – Altitude classes from the DME source and estimated polynomial for $r=s= 20$.

Altitude Classes(m)	Total points from de DME source data	Total points from estimated polynomial
< 100	227	66646
0101 - 200	4828	12079
0201 - 300	13674	13849
0301 - 400	35192	28908
0401 - 500	33607	51390
0501 - 600	94169	68011
0601 - 700	169517	121438
0701 - 800	80245	100917
0801 - 900	89571	87524
0901 – 1000	136910	70732

1001 – 1100	18658	32397
1101 – 1200	17588	15420
1201 – 1300	20127	9097
1301 – 1400	2207	7251
1401 – 1500	1923	5543
1501 – 1600	2559	4280
1601 – 1700	406	3513
1701 -1800	405	2930
1801 -1900	281	2531
1901 – 2000	50	2156
2001 – 2100	50	1894
2101 – 2200	49	1654
2201 – 2300	15	1433
2301 – 2400	9	1250
2401 – 2500	10	1098
2501 – 2600	6	956
2601 – 2700	7	873
2701 – 2800	3	755
2801 – 2900	1	695
> 2900	0	5074
Minimal Altitude	1	-4374
Maximum Altitude	2863	5606

5. Conclusion

The adjusted polynomial of degrees 20 in x and 20 in y has 441 coefficients. If we use the type float to store them, it will be necessary 1.764 Kb of storage. The image, on the other hand, with 722.294 points, requires at least 2 bytes to indicate the altitude of each point, being necessary the total of 1.444 Mb to store it. The adjusted polynomial needs about 0.1221% of space used by the image. The adjusted coefficients are shown on **Appendix A**.

From the results it is verified that is possible to represent, satisfactorily, the Landform Attributes of an area by a high degree polynomial and the representation has the advantage of smaller space. Additionally, the functional representation of the Landform Attributes allows larger efficiency, in time and space when sending this information through networks. Efficiency in time is real since, instead of sending an image with million of points, the coefficients of a mathematical function are sent. Efficiency in space since, instead of storing the image, we may store the coefficients that represents it.

In this presented methodology, as larger as the degree of the polynomial, better the estimate. For this work a polynomial of degree 20 was used, however with a polynomial of degree 40 or larger, the results, statistically, would be better.

The presented methodology also has the advantage of being easily adaptable for distributed processing, where the necessary processing to estimate each parameter can be a task for a network machine.

6. Referências Bibliográficas

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Appendix A - Adjusted parameters a_{ij} for the polynomial of degree 20

i	j	a_{ij}
0	0	-600958.0
0	1	5782.95
0	2	-19.4891
0	3	0.0288535
0	4	-1.90105e-005
0	5	1.00907e-008
0	6	-1.08824e-011
0	7	-5.82749e-016
0	8	9.41411e-021
0	9	2.12736e-020
0	10	-1.29512e-023
0	11	-5.39167e-027
0	12	-3.27327e-030
0	13	3.1375e-033
0	14	7.41835e-036
0	15	-2.22569e-039
0	16	-2.76506e-042
0	17	4.15468e-046
0	18	-1.27859e-048
0	19	2.23836e-051
0	20	-7.66139e-055
1	0	-1803.36
1	1	3.49567
1	2	0.0102505
1	3	-3.18755e-005
1	4	1.9612e-008
1	5	2.46465e-011
1	6	-5.56414e-014
1	7	4.63612e-017
1	8	-1.89213e-020
1	9	-1.5968e-023
1	10	8.15077e-026
1	11	-8.06989e-029
1	12	-1.59881e-032
1	13	5.07274e-035
1	14	-1.66356e-038
1	15	2.35144e-042
1	16	-1.59586e-045
1	17	4.21912e-048
1	18	-3.8003e-051
1	19	-2.08191e-054
1	20	1.74265e-057
2	0	17.2571
2	1	-0.0798998
2	2	0.000136495
2	3	-9.87725e-008
2	4	3.22612e-011
2	5	-4.74183e-014
2	6	1.03617e-016
2	7	-4.59738e-020
2	8	-5.36129e-023
2	9	-3.28087e-026
2	10	1.30999e-028

2	11	-9.96481e-032
2	12	2.32681e-035
2	13	2.91191e-038
2	14	1.98463e-041
2	15	-3.89582e-044
2	16	1.27689e-049
2	17	-2.03917e-050
2	18	2.28209e-053
2	19	1.31291e-056
2	20	-9.58082e-060
3	0	-0.0308943
3	1	0.00010274
3	2	-1.27336e-007
3	3	3.64345e-011
3	4	-5.16296e-014
3	5	7.03445e-017
3	6	9.37562e-020
3	7	-6.68183e-023
3	8	-2.03602e-026
3	9	9.50303e-030
3	10	-3.92312e-032
3	11	1.15359e-034
3	12	-1.5896e-037
3	13	7.84564e-041
3	14	-2.41806e-044
3	15	1.94771e-047
3	16	-1.00101e-050
3	17	2.23836e-054
3	18	1.34827e-056
3	19	-4.67469e-059
3	20	1.62563e-062
4	0	3.01785e-005
4	1	5.62669e-008
4	2	-1.24528e-010
4	3	1.41046e-013
4	4	8.3755e-017
4	5	-3.39007e-019
4	6	6.38611e-023
4	7	-6.92627e-027
4	8	8.84677e-029
4	9	-5.60388e-032
4	10	7.09316e-035
4	11	8.91026e-038
4	12	-2.72485e-041
4	13	1.57051e-045
4	14	-8.24462e-047
4	15	-4.01016e-050
4	16	1.54209e-052
4	17	-7.07798e-056
4	18	6.53703e-059
4	19	-7.20566e-063
4	20	9.03631e-066
5	0	-9.9277e-008
5	1	-1.3463e-010

5	2	1.02058e-013
5	3	4.3139e-017
5	4	6.68576e-020
5	5	1.08416e-023
5	6	-8.79302e-027
5	7	5.00884e-029
5	8	-7.8897e-032
5	9	-5.9284e-035
5	10	-1.21184e-037
5	11	3.09327e-041
5	12	-3.72746e-044
5	13	2.28254e-047
5	14	4.2481e-050
5	15	-1.918e-053
5	16	1.07273e-056
5	17	-9.64576e-059
5	18	4.49445e-062
5	19	-9.4825e-065
5	20	3.29946e-069
6	0	2.41114e-010
6	1	-6.5032e-014
6	2	8.6818e-017
6	3	-1.02433e-019
6	4	-5.17244e-023
6	5	1.47669e-025
6	6	4.49713e-030
6	7	-3.4049e-032
6	8	1.86206e-034
6	9	1.15639e-037
6	10	-1.29643e-040
6	11	6.52723e-044
6	12	1.06523e-046
6	13	-7.80133e-050
6	14	-4.82181e-053
6	15	1.75495e-056
6	16	-6.36137e-059
6	17	7.16125e-062
6	18	-3.41804e-065
6	19	1.38106e-067
6	20	-1.22422e-071
7	0	-1.84225e-013
7	1	2.71599e-016
7	2	-3.23003e-019
7	3	7.60932e-023
7	4	-8.98206e-026
7	5	-1.08465e-028
7	6	-3.00545e-032
7	7	1.67546e-034
7	8	-8.46936e-038
7	9	-4.59146e-041
7	10	-3.2693e-044
7	11	-7.98322e-047
7	12	9.65665e-050
7	13	-1.60805e-052

7	14	2.5748e-055
7	15	3.15653e-059
7	16	-2.14649e-062
7	17	9.76311e-065
7	18	-4.42692e-068
7	19	-8.76675e-071
7	20	9.27286e-075
8	0	-9.61553e-017
8	1	-9.04873e-020
8	2	1.71494e-022
8	3	2.06979e-027
8	4	-1.27143e-028
8	5	-4.01233e-032
8	6	-4.98303e-035
8	7	3.40135e-038
8	8	-4.11776e-040
8	9	1.43717e-044
8	10	-3.3553e-047
8	11	2.73705e-049
8	12	2.56637e-053
8	13	-3.53254e-055
8	14	1.61812e-058
8	15	-1.10237e-061
8	16	4.98723e-065
8	17	-1.0581e-067
8	18	8.75835e-071
8	19	-1.05405e-073
8	20	7.55954e-077
9	0	1.8615e-019
9	1	-1.60112e-022
9	2	6.38252e-027
9	3	-2.61119e-030
9	4	1.95847e-031
9	5	2.76744e-034
9	6	-1.65095e-037
9	7	4.02441e-040
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